Grade 4: Distributive Multiplication Algorithm

4.N.6	
Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products.	1. Model a given multiplication problem using the distributive property, e.g., 8 x 365 = (8 x 300) + (8 x 60) + (8 x 5). 2. Use concrete materials, such as base ten blocks or their pictorial representations, to represent multiplication and record the process symbolically. 3. Create and solve a multiplication problem that is limited to 2- or 3-digits by 1-digit. 4. Estimate a product using a personal strategy, e.g., 2 x 243 is close to or a little more than 2 x 200, or close to or a little less than 2 x 250. 5. Model and solve a given multiplication problem using an array and record the process. 6. Solve a given multiplication problem and record the process.

Clarification of the outcome:

✦ This concerns 1-digit x 2 or 3-digit multiplication, using an algorithm that make sit relatively easy to see why it works. The traditional multiplication algorithm that has been taught for many years is not well suited to helping students understand why it works.

✦ <u>Critical</u> to understanding the multiplication algorithm is <u>understanding the distributive principle</u>. It is this principle that has future payoff in other mathematical arenas. The algorithm itself is only useful for doing arithmetic.

Required close-to-at-hand prior knowledge:

✤ Automaticity of most multiplication facts.

✤ Distributive principle (to be developed prior to/along with the algorithm).

✤ Multiplication by 10s and 100s.

✤ 2 and 3-digit addition.

SET SCENE stage

Ask students to describe situations where you follow a script and/or that repeat. Situations such as following a recipe (following a script) and sliding down a hill, walking up, sliding down again, and so on (repeat) are some possibilities.

Tell them that a machine needs to be given instructions on how to multiply "big" numbers (such as 8 x 239). Their job will be to tell the machine what to do. Of course, first they need to understand how to multiply big numbers themselves.

The problem task to present to students:

Organize students into groups. Tell them that the machine first has to know how to multiply by tens. Ask each group to figure out a way of multiplying 1 x 10, 2 x 10, 3, x 10, . . . 9 x 10.

Provide PV materials for them to use.

Comments:

An algorithm is a series of steps (with repetitive qualities) that is used to do some task. In the case of arithmetic, an algorithm is a way of obtaining answers to arithmetic tasks.

Students need to understand that efficient algorithms are useful because those algorithms can free the mind for dealing with more complicated matters. The crucial point is though, the algorithm has to be understood. It should not be a mindless procedure students do because someone told them what to do.

DEVELOP stage

This lesson plan assumes students do not know how to multiply efficiently by 10s and 100s and that they do not understand the distributive principle.

Accordingly, the plan consists of many activities and might take about 2 weeks to complete but that time is well worth it.

Activity 1: Revisits SET SCENE and addresses achievement indicators 2 and 6.

Ask selected groups to present their method for multiplying by tens. [Some my suggest repeated addition; e.g. 3 x 10 is 10 + 10 + 10. Some may suggest skip counting. And so on.] Ensure students understand why each method works. Accept each method but then steer them to the point of view that we need a fast method for the machine to use because that it saves time and energy. Guide students to a shortcut by using patterning. For example, have them notice what what happens for 1 x 10 = 10, 2 x 10 = 20, 3 x 10 = 30 , . . . They should see the pattern of sticking one zero on the end of the multiplier digit.

Activity 2: Addresses achievement indicators 2 and 6.

- Tell students that the machine now needs to know how to multiply by 20, 30, . . . 90. Organize students into groups. Ask each group to figure out a way of multiplying a <u>single digit</u> by 20, 30, . . . 90. Provide PV materials for them to use.
- Ask selected groups to present their method for multiplying. Accept each method but then steer them to the point of view that once again we need a fast method for the machine to use. Guide students to a shortcut by using patterning as before. [Note: The bugaboo in this is when an extra zero occurs, as in for example 5 x 20. Ensure students realize that you still multiply the single digit times the tens digit of the tens number and then stick on a zero at the end.]

Activity 3: Addresses achievement indicators 2 and 6.

Repeat activities #1 and #2 but this time the matter-at-hand is: (1) multiplying a single digit by 100s and (2) multiplying a single digit by 200, 300, . . . 900. Ensure students realize the shortcut now is to stick on two zeros at the end.

Activity 4: Practice on x 10s and x 100s and addresses indicator 4.

- Provide practice on multiplying a <u>single digit</u> by 10s and 100s. A basic worksheet is useful. It should include such questions as 5 x 10, 7 x 100, 3 x 40, 8 x 900.
- Put a list of ten questions such as 5 x 38, 5 x 32, 6 x 391, 6 x 312 on the board. Ask students how we can estimate an answer to such questions. Ensure they realize a good way is to think of, for example, 6 x 391 as 6 x 400. Have students estimate answers.

Activity 5: Addresses achievement indicators 2, 3, and 6.

Ask students to provide some examples of the multiplication the machine can do right now. Expect x 10s and x 100s and examples such as 4 x 20 = 80 and 4 x 300 = 1200.

Present students with a problem involving multiplication of a 1-digit number times a 3-digit number where the 3-digit number is not a multiple of a hundred. [For example, "*Mary is exactly 5 years old. She wonders how many days she is old. Help Mary solve her problem.*"] Ask students to represent the problem by writing a multiplication number sentence. In the case of the example, they should write 5 x 365 = ? (5 groups of 365 days is ?).

Ask students to obtain the answer to the multiplication in some way. [They will likely use repeated addition and/or PV materials.] Discuss how much work it is to obtain the answer by repeated addition and/or using place value materials. Suggest that once again the machine needs a faster way. Tell students that we first need to look at simpler multiplication tasks to figure out a faster way.

Activity 6: Addresses achievement indicators 2, 5, and 6.

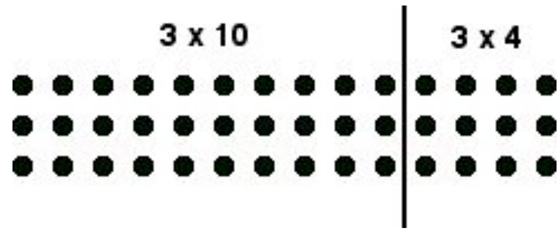
Tell students that the machine needs to know how rows and columns can show multiplication.

Have groups of students play the game, 'Draw rectangles'. The game requires a score sheet, grid paper, a three minute timer, and two six-sided dice, having the numbers 1 to 6 on each of them. Have at most three students per group.

- ✦ Two groups play each other. Decide which group goes first by having each group throw the two dice. The highest PRODUCT showing goes first. To play the game, group #1 throws the two dice and multiplies the two numbers showing together. The members of the group then have three minutes to draw all the rectangles they can think of that have dimensions whose product is the same as the product showing on the two dice but where the none of the dimensions is 1. For example, if 5 and 6 show up on the dice, the product is 30. Group #1 cannot draw rectangles that are 1 x 30 or 30 x 1 but they can draw rectangles that are 5 x 6, 6 x 5, 2 x 15, etc. When the time is up or group #1 cannot draw any more rectangles, group #1 receives one point for each correct rectangle. Then it is group #2's turn to throw the dice and to draw rectangles. The two groups take turns until one of them obtains a total of 6 points or more.
- ✦ When the game play is over, ask students to explain why a rectangle is a way of showing multiplication. Ask students how many dots are needed inside a 5 x 6 rectangle to show the answer to 5 x 6. Ask how many dots are needed inside a 8 x 100 rectangle to show the answer to 8 x 100.

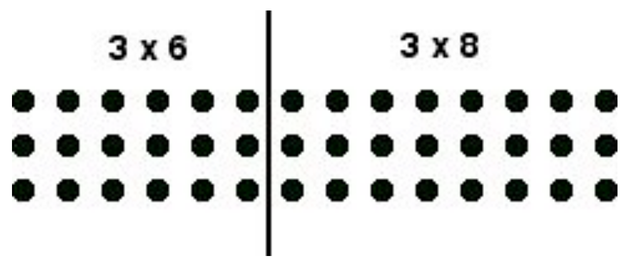
Activity 7: Addresses achievement indicators 1, 2, 3, 5, and 6 for 1-digit x 2-digit.

Present the question 3 x 14. Have them represent the multiplication, using an array of dots, and then have them split the long side of the array along place value lines. Ask them to write in each smaller array the multiplication that it represents. Assist them as needed. When these tasks are completed, the result should look like as shown here.



Discuss if we can obtain an answer to 3 x 14 by doing 3 x 10 and 3 x 4 and then adding the two multiplication results together. Confirm that the method works by having them use repeated addition to do 3 x 14.

Ask them whether it might be possible to split the long side in a different way, for example, split 14 into 6 and 8. Have them draw the dot array for that way of spitting 14. The result should look as shown here.

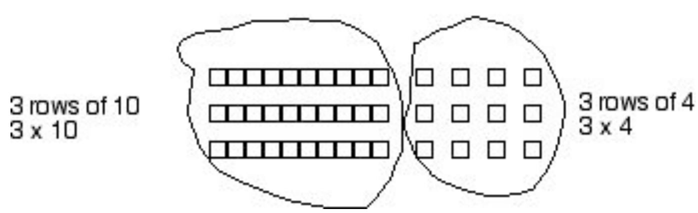


Have them obtain the answer for 3 x 14 by doing 3 x 6 add 3 x 8.

Have them split 14 in other ways and obtain the answer to 3 x 14 each time. [Why? The reason is to develop a functional sense of the distributive principle (see note #2 on next page).]

Discuss why it might be faster to obtain an answer to 3 x 14 by splitting 14 along place value lines (10 + 4) instead of splitting 14 in other ways (e. g. 6 + 8). Ensure that students realize that spitting up along place value lines and multiplying by 10 is simpler to do, and therefore should be faster, especially when numbers get bigger.

Present students with at least four more 1-digit x 2-digit multiplication questions. Restrict the 2-digit numbers to less than 40. Have students draw dot diagrams (or use place value materials - see the example here) and split the 2-digit number along place value lines to obtain answers . Ensure that their diagrams match the multiplications and additions.



Have them record the concrete situations symbolically, using brackets (mathematically not needed but brackets visually highlight the groupings). For example, for 3 x 14, assist them to record the dot diagram (or place value blocks situation) as: 3 x 14 = (3 x 10) + (3 x 4).

Note 1:

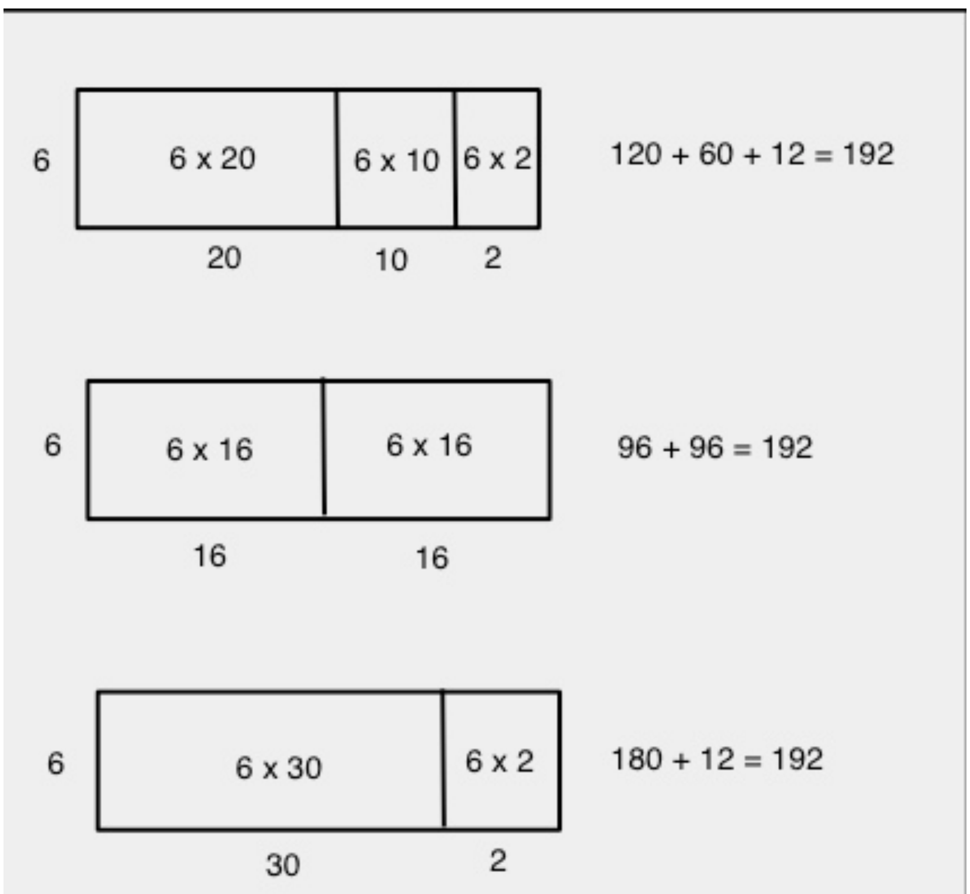
Using place value blocks can be cluttered. For example, to show 3 x 40, you would have to have 3 rows of four tens (the longs), where the four tens are stacked on top of each other. You cannot show 3 x 40 as 12 x 10 because the question concerns 3 x 40.

Note 2:

Activity #7 develops the distributive principle without giving exposing students to the jargon - distributive.

Students should understand the functional sense of the distributive principle:

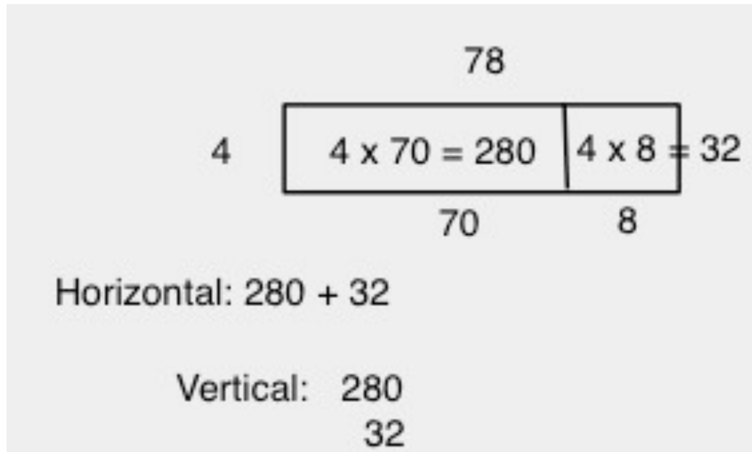
- You can split up a multiplication into smaller products and then add the smaller products to get the answer to the multiplication. The splitting can be done in many ways. Usually slitting along place value lines leads to the easiest way to obtain the answer. The diagram below illustrates a variety of ways to split 32 to obtain the answer to 6 x 32 when using the distributive principle.



Activity 8: Addresses achievement indicators 1, 2, 3, 5, and 6 for 1-digit x 2-digit.

Present six questions such as 5 x 46, 7 x 92, 3 x 15, 2 x 87, 4 x 73, and 9 x 62. Ask students to describe the instructions for the machine so that it can do the multiplication. Ensure they realize breaking up the 2-digit number along place value lines is the easiest instruction for the machine, and the one that leads to a fast way to do the multiplication.

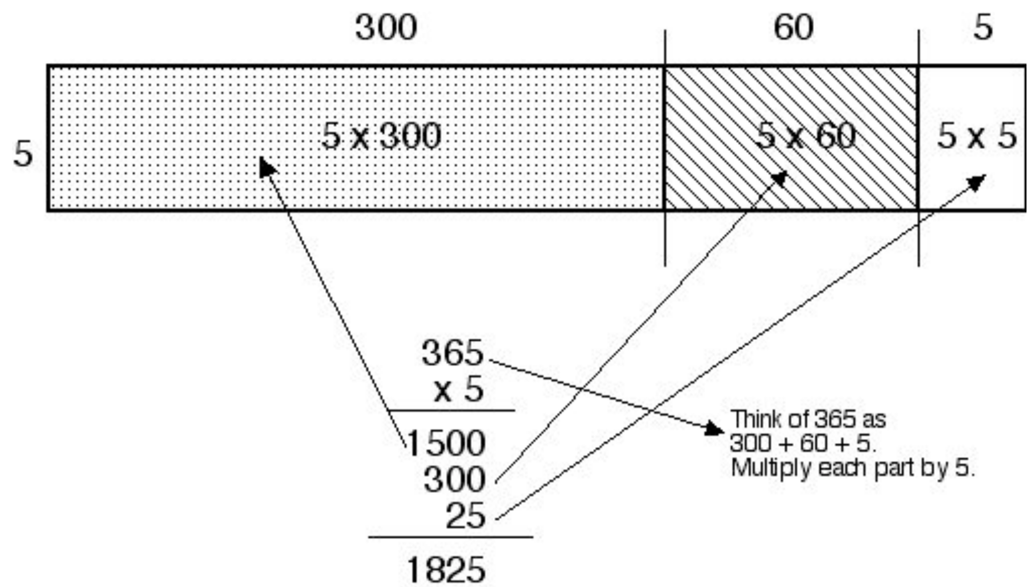
Refer to the game of activity #6. Discuss whether it would be "fun" to draw all of the dots. Suggest an easier way to show a multiplication question such as 4 x 78 is by drawing a rectangle with the dimensions of the rectangle representing the dots. Ask how the rectangle diagram could be used to break up the multiplication into smaller parts. Ensure they understand what is shown here.



Have students obtain answers to the six multiplication questions by drawing a rectangle diagram, breaking up the 2-digit number along place value lines, obtaining the smaller products, and then adding them together to get the final product. Ask students to do the addition horizontally and also vertically.

Activity 9: Addresses achievement indicators 1, 2, 3, 5, and 6 for 1-digit x 3-digit.

Present six questions such as 5 x 365, 7 x 902, 3 x 115, 2 x 807, 4 x 73, and 9 x 125. Ask students how the answer to 5 x 365 (for example) can be done by using rectangles and breaking up the 3-digit number along place value lines. Ensure they understand what is shown here. The addition should be done vertically (shown in the diagram) and horizontally (not shown in the diagram).



Have students obtain answers to the remaining questions by drawing a rectangle diagram, breaking up the 3-digit number along place value lines, obtaining the smaller products, and then adding them together to get the final product. Ask students to do the addition horizontally and also vertically.

Activity 10: Addresses achievement indicators 1, 2, 3, 5, and 6 for 1-digit x 3-digit.

Return to the question from activity #5. [For example: "*Mary is exactly 5 years old. She wonders how many days she is old. Help Mary solve her problem.*"]

- ✦ Ask students to describe the instructions for the machine so that it can do 5 x 365. Have students obtain the answer to 5 x 365.
- ✦ Discuss with students how they could figure out how old they are exactly in days. Ensure they realize to do that they would have to multiply their age in years and add on the number of days past their birthdate. Have students figure out their age in days. Record the results and save them for a later activity involving bar graphs where the horizontal axis would have to be in intervals (for example, 4000 to 4050 days, etc).

Activity 11: Practice.

Write the following multiplication questions on the board:

- 6 x 10
- 7 x 30
- 5 x 40
- 8 x 100
- 6 x 700
- 5 x 800
- 4 x 38
- 7 x 234

Ask students to describe the instructions for the machine so that it can do the listed multiplication questions.

Activity 12: Practice.

Provide students with about three multiplication word problems that concern 1-digit x 2-digit multiplication. [E.G. *Harry has 6 boxes with 31 candies in each box. How many candies does Harry have in the boxes?*] Ask students to estimate answers. Discuss their method of estimating and the results. [For example, 6 x 31 is like 6 x 30 and that is 180.] Then have students figure out the exact answers and compare them to the estimates.

Provide students with about three multiplication word problems that concern 1-digit x 3-digit multiplication. [E.G. *Harry has 8 pages with 271 stamps ion each page. How many stamps does Harry have on the pages?*] Ask students to estimate answers. Discuss their method of estimating and the results. [For example, 8 x 271 is like 8 x 300 and that is 2400. Then have students figure out the exact answers and compare them to the estimates.

Activity 13: Assessment of teaching.

Provide a worksheet that consists of four multiplication questions: (1) a 1-digit x 2-digit multiplication, (2) a 1-digit x 3-digit multiplication, (3) a 1-digit x 2-digit multiplication, and (4) a 1-digit x 3-digit multiplication.

Have students do questions #1 and #2 by drawing a rectangle diagram showing where each subproduct comes from and by using the vertical and horizontal adding method. For questions #3 and #4, ask students to estimate an answer and explain how they did the estimate.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

Two examples of partially well-designed worksheets follow.

Each worksheet contains a sampling of question types. More questions of each type are needed for a well-designed worksheet.

The MAINTAIN stage follows the sample worksheets.

Question 1

4 x 30 = _________ 2 x 50 = _________ 7 x 200 = _________ 5 x 400 = _________

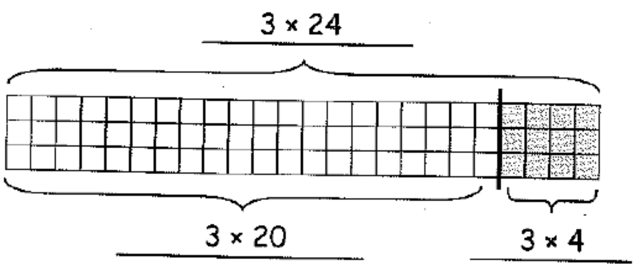
Question 2

Knowing that 3 x 2 = 6, what is 3 x 2000? ____________________________

Question 3

Write a multiplication statement for the whole array and each part of the array (as shown in a).

a)



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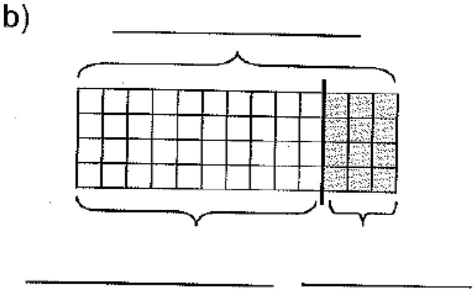

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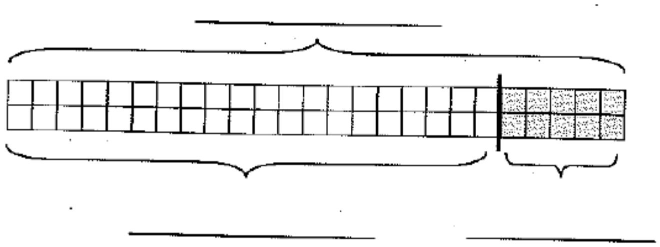

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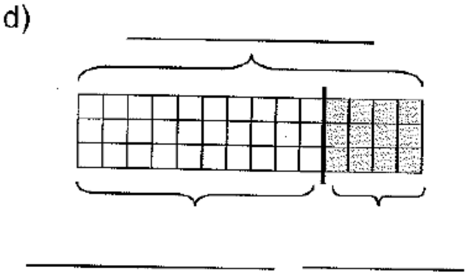


Question 4

Multiply, using the method shown in a.

a. 3 x 13 = 3 x 10 + 3 x 3 = 30 + 9 = 39
b. 5 x 14 = _______ + ______ = ______ = ______
c. 6 x 35 = _______ + ______ = ______ = ______
d. 8 x 71 = _______ + ______ = ______ = ______

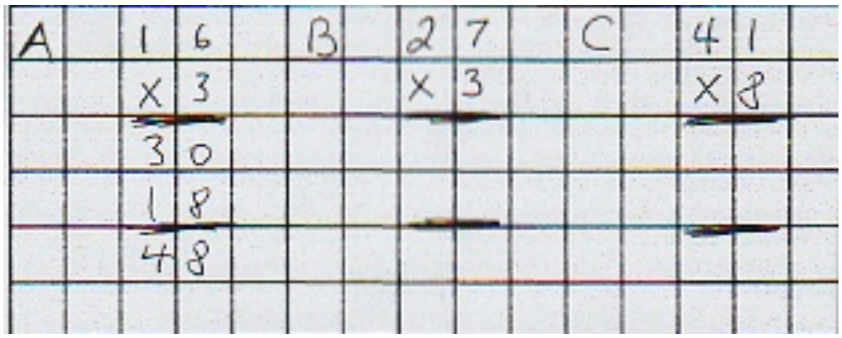
Question 5

Multiply, using the method shown in a.

a. 3 x 142 = 3 x 100 + 3 x 40 + 3 x 2 = 300 + 120 + 6 = 426
b. 5 x 168 = ________ + ________ + _____ = ______________ = ______
c. 6 x 357 = ________ + ________ + _____ = ______________ = ______
d. 8 x 713 = ________ + ________ + _____ = ______________ = ______

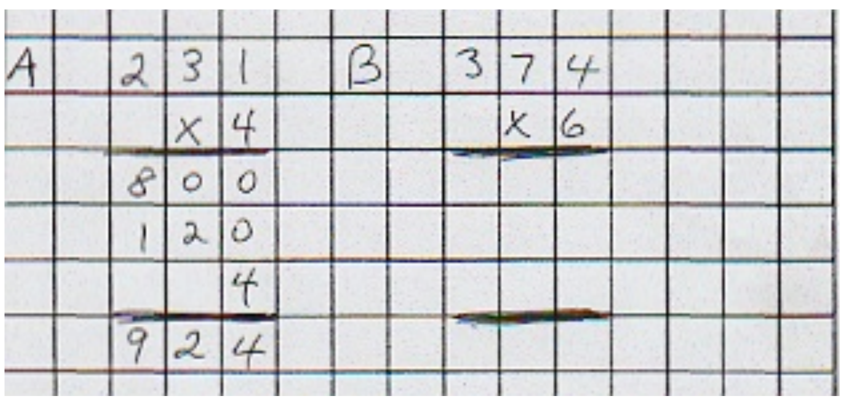
Question 6

Multiply, using the method shown in A.



Question 7

Multiply, using the method shown in A.



Question 7

Solve the problem:

Yen planted 23 trees in each of 5 rows. How many trees did she plant?

Question 8

Solve the problem:

Paul put 240 marbles in each of 3 bags. How many marbles did he put in the bags?

MAINTAIN stage

Mini-task example

Every once in a while, put ONE 1-digit x 3-digit multiplication question on the board. Ask students to estimate the answer and to figure out the exact answer. For the exact answer, they must also explain the algorithm (using words and/or a drawing).

Rich-task example

Have students solve the following problem.

An electronic games store receives shipments of Nintendo games in cartons having 24 games in each carton. It receives shipments of Game Boy games in cartons having 144 games in each carton. On Monday, the store received 7 boxes of Nintendo games and 8 boxes of Game Boy games. How many games did the store receive on Monday?

Comments.

This is a rich-task because it involves a complex word problem.